AS

Mathematics

MPC2 – Pure Core 2
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

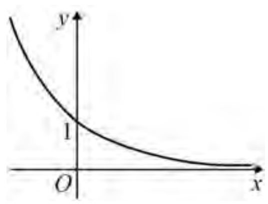
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\frac{1}{x^2} = x^{-2}$	B1		$\frac{1}{x^2} = x^{-2}$ seen, or used. PI by correct integration of $\frac{36}{x^2}$.
	$\int \left(\frac{36}{x^2} + ax\right)\,dx = \left[-\frac{36}{x} + \frac{ax^2}{2}\right]$ $(+k)$	M1		Correct integration of either $\frac{36}{x^2}$ or ax.
		A1	3	Correct integration of both terms, ACF, accept unsimplified. Condone missing $+k$.
(b)	$\int_1^3 \left(\frac{36}{x^2} + ax\right)\,dx =$			
	$\left(-\frac{36}{3} + \frac{9a}{2}\right) - \left(-36 + \frac{a}{2}\right)$	M1		Attempt to find $F(3) - F(1)$, following attempt at integration . (**M0** if $F(x) = \frac{36}{x^2} + ax$ or if uses an $F(x)$ which contains part of the given integrand)
	$24 + 4a = 16$, $(a =) -2$	A1	2	-2 **NMS** scores 0/2
	Total		5	
(b)	Using the correct answer to (a), the example $-\frac{36}{3} + \frac{9a}{2} + 36 + \frac{a}{2}$ would get the M1 in (b) if no further evidence suggested the cand was working with F(3)+F(1).			

Q2	Solution	Mark	Total	Comment
(a)		B1		Only one y- intercept, marked as 1 or coordinates (0,1) stated or '$y = 1$ when $x = 0$'
		B1		Correct graph having no other 'crossing point' on either axis.
			2	
(b)	$x \log 0.2 = \log 4$	M1		OE eg $(x =) \log_{0.2} 4$
	$(x =) -0.861(35...)$ $= -0.861$ (to 3sf)	A1	2	Condone > 3sf, rounded or truncated. If use of logarithms not explicitly seen then score 0/2
(c)	Reflection in the y-axis.	E1	1	OE **E0** if more than one transformation
	Total		5	
(a)	For large positive x-values, graph in 1st quadrant, if not very close to the x-axis, must be approaching the horizontal which is clearly closer to the x-axis than through c's y-intercept			
(b)	OEs eg $-x\log 5 = \log 4$; $x \log 2 = \log 4 + x \log 10$; $x(1 - \log_2 10) = 2$; $(x-2)\log 2 = x \log 10$			

Q3	Solution	Mark	Total	Comment
(a)	$\left(\frac{dy}{dx}=\right)\ \frac{6}{2}x^{-0.5}-1\ =\ 3x^{-0.5}-1$	B2,1	2	ACF. If not B2, award B1 for correct differentiation of either $6x^{1/2}$ or $-x-3$
(b)	$3x^{-0.5}-1=0$	M1		Evidence of c's $\frac{dy}{dx}$ equated to 0 to form an equation in *x*.
	$3x^{-0.5}=1$, $x=9$	A1F		Only ft if c's $\frac{dy}{dx}=ax^{-0.5}-1$ ie $x_M=a^2$
	(*y*-coordinate of *M* is) 6	A1	3	NMS scores 0/3
(c)	At *P*(4,5) $\frac{dy}{dx}=3(4)^{-0.5}-1$ (=0.5)	M1		Attempt to find c's $\frac{dy}{dx}$ when $x = 4$.
	Gradient of normal = -2	m1		$m \times m' = -1$ used
	Eqn of normal $y-5=-2(x-4)$	A1	3	ACF eg $y+2x=13$
(d)	Translated normal: $y-5=-2(x-k-4)$	M1		Either $x \to x-k$ or $x \to x+k$ with no change to *y* in cand's eqn of normal seen or used
	Passes through *M* (9, 6) so $6-5=-2(9-k-4)$	m1		Subst of c's *M* coordinates (both +'ve) into cand's eqn of normal with $x \to x-k$ and no change to *y*.
	$k = 5.5$	A1	3	A correct value of *k* with no errors seen.
ALTn 1	On normal, when *y*=6, $6-5=-2(x-4)$	(M1)		Sub answer **(b)** in answer **(c)**: ie attempt to find x_N, the *x*-coordinate of point on c's normal in part (**c**) with same *y*-coord as c's part **(b)** answer.
	$x = 3.5$; $3.5+k=x_M=9$	(m1)		(c's x_N) $+k$ = (c's x_M) provided c's x_M is >0
	$k = 5.5$	(A1)	(3)	A correct value of *k* with no errors seen.
ALTn 2	Line through *M* parallel to normal at *P* has equation $y-6=-2(x-9)$	(M1)		Correct ft eqn using c's *M* coords, both>0 and c's normal at *P*
	eg Using *y*=0, for normal at *P*, (6.5, 0) and for parallel line through *M*, (12,0) (*k* =) $12-6.5$	(m1)		For any single value of *y*, finding the *x* coord of the point on both normal at *P* and this parallel line through *M* and then subtracting these *x* coords in correct order
	$k = 5.5$	(A1)	(3)	A correct value of *k* with no errors seen.
	Total		**11**	

Q4	Solution	Mark	Total	Comment
(a)	[S_{21}=] $\frac{21}{2}\left[2a+(21-1)d\right]$	M1		$\frac{21}{2}\left[2a+(21-1)d\right]$ OE
	$\frac{21}{2}\left[2a+20d\right]$=168	m1		Forming correct eqn
	21(a+10d)=168 $\Rightarrow$ a + 10d = 8	A1	3	AG a + 10d = 8 convincingly obtained with intermediate step shown eg 21(2a+20d)=168×2; 2a+20d=8×2
(b) (i)	$a+d+a+2d=50$ $(2a+3d=50)$	M1		$a+d+a+2d=50$ OE in terms of a and d
	$2(8-10d)+3d=50$	m1		Solving a + 10d = 8 OE simultaneously with c's 2a + 3d = 50 OE as far as correctly eliminating either a or d. PI by correct values for both d and either a or u_{12}.
	$d=-2$; a = 28 or 12th term = 8+d	A1		$d=-2$ **and** either a = 28 or 8+d seen or used in part **(b)(i)**.
	$(u_{12}=)$ 6	A1	4	**NMS** scores 4/4 unless FIW
(b)(ii)	$\sum_{n=4}^{21}u_n=\sum_{n=1}^{21}u_n-\sum_{n=1}^{3}u_n$	M1		$\sum_{n=4}^{21}u_n=\sum_{n=1}^{21}u_n-\sum_{n=1}^{3}u_n$ OE eg $S_{21}-S_3$ stated or used
	= 168 – (a + 50) or 168 – 1.5(2a + 2d)	A1F		OE. If numerical form only then ft on c's non-zero values for a and d.
	= 90	A1	3	90 NMS 90 scores 3/3 unless FIW. **SC** If 0/3 award 1 mark for answer 68
	Altn. $A=a+3d$, N=18,			
	$\sum_{n=4}^{21}u_n=\frac{18}{2}\left[2(a+3d)+(18-1)d\right]$	(M1)		OE Seen or used. {OEs include 9(2a+23d) =18a + 207d}
	$=\frac{18}{2}\left[2(a+3\times(-2))+(18-1)(-2)\right]$	(A1F)		Ft on c's $d\neq0$ value in **(b)(i)**; if expression is entirely numerical, also ft on c's $a\neq0$ value
	$\sum_{n=4}^{21}u_n$ = 90	(A1)	(3)	90
	Total		**10**	

Q5	Solution	Mark	Total	Comment
(a)	$h = 3$	**B1**		$h = 3$ OE stated or used. (PI by x-values 2, 5, 8, 11 provided no contradiction)
	$f(x) = \sqrt{x^2+9}$ $I \approx \frac{h}{2}\{f(2)+f(11)+2[f(5)+f(8)]\}$	**M1**		$h/2\{f(2)+f(11)+2[f(5)+f(8)]\}$ seen or used OE summing of areas of the 'trapezia'.. (**M0** if using an incorrect f(x))
	$\frac{h}{2}\text{with}\{\} = \sqrt{13}+\sqrt{130}+2\left(\sqrt{34}+\sqrt{73}\right)$ $= \frac{h}{2}\{$ 3.60(5..)+11.4(0..)+ +2[5.83(09..)+8.54(4..)]} $= \frac{h}{2}\{$ 15.0(07..)+28.7(499..)]} ($I \approx 1.5 \times 43.7(57..)$) (= 65.63(58…)	**A1**		OE Accept 3sf or better evidence for surds. Can be implied by later <u>correct</u> work provided >1 term or a single term for I which rounds to 65.6
	I = 65.6 (to 1 dp)	**A1**	**4**	CAO Must be 65.6 **SC** 4 strips used:**Max** B0M1A0; 65.5 A1
(b)(i)	Translation $\begin{bmatrix}0\\5\end{bmatrix}$	**E2,1,0**	**2**	**E2**: 'translat…' and $\begin{bmatrix}0\\5\end{bmatrix}$ ... If not **E2** award **E1** for 'translat… in y-dir' OE. **E0** if more than one transformation
(b)(ii)	Stretch (**I**) in x-direction (**II**) scale factor $\frac{1}{3}$ (**III**)	**E2,1,0**	**2**	Need (**I**) and (**II**) and (**III**) for **E2.** If not **E2** then award **E1** for seeing $3\sqrt{x^2+1} = \sqrt{(3x)^2+9}$ OE together with (**I**) and either (**II**) or (**III**) **E0** if more than one transformation
	Total		**8**	
(a) **(b)(i)** **(b)(ii)**	For guidance, separate trapezia 14.1(5…)+21.5(6…)+29.9(18..) NB For E2 the vector must be written in column format Accept 'horizontal…' OE for 'x-direction'			

Q6	Solution	Mark	Total	Comment
(a)	$5^2 = 8^2 + 9^2 - 2(8)(9)\cos\theta$	M1		Cosine rule used correctly. Accept eg A for θ if intention is clear. PI by next line
	$\cos\theta = \dfrac{8^2 + 9^2 - 5^2}{2(8)(9)} \quad \left(= \dfrac{120}{144} = \dfrac{5}{6}\right)$	m1		Allow one sign slip in rearrangement from a correct M1 line
	$\theta = 0.5856(855..)$ = 0.586 to 3sf	A1	3	AG Must see more than 3sf for the angle before seeing printed value 0.586
(b)	(Area of triangle) = $\dfrac{1}{2} \times 9 \times 8 \sin\theta$	M1		OE Correct value or correct expression involving no unknown values. eg $\sqrt{11(11-9)(11-8)(11-5)}$
	$= 36\sin\theta$ = 19.9 (cm^2)	A1		If >3sf accept a value from 19.89 to 19.91 inclusive.
			2	NMS: Award 2 marks for 19.9 or 'better'
(c)	(Area of sector =) $\dfrac{1}{2}r^2\theta$	M1		$\dfrac{1}{2}r^2\theta$ seen, or used, for sector area
	$\dfrac{1}{2}r^2\theta = 36\sin\theta - \dfrac{1}{2}r^2\theta$	m1		OE Ft c's answer to **(b)** if incorrect eg $\dfrac{1}{2}r^2\theta = \dfrac{\text{"19.9"}}{2}$
	$r^2 = \dfrac{36\sin\theta}{\theta} \approx 34, \quad r = 5.83$	A1		PI by later work. Accept 5.82 to 5.83 inclusive.
	(Arc length=) $r\theta$	M1		$r\theta$ seen, or used, for arc length
	Perimeter (of shaded shape) $= r\theta + 5 + 8 - r + 9 - r$	m1		Correct expression, seen or used, for the required perimeter.
	$= 22 + r(\theta - 2)$			
	= 13.8 (cm) to 3sf	A1	6	CAO must be 13.8
	Total		**11**	
(a)	Accept 0.5856 or 0.5857 or AWFW 0.5856 to 0.5857 as evidence for the A1			
(b)	NMS: 'better' means 'value from 19.89 to 19.91 inclusive'			

Q7	Solution	Mark	Total	Comment
(a)	$p = -10$; $q = 40$; $r = -80$	**B1** **B1** **B1**	 **3**	Accept correct embedded values for p, q and r within the expansion
(b)	$(2+x)^7 = \ldots\ldots\ldots + mx^5 + nx^6 + x^7$	**M1**		Attempting to find **at least two of** x^5 term , x^6 term , x^7 term in the expansion of $(2+x)^7$
	$m = 84$, $n = 14$	**A1**		Either correct. (M1 must be scored). PI by later correct work
	Coefficients of x^{10} terms in expansion of $(1-2x)^5(2+x)^7$ are $-32m+80n+r$	**m1**		Identifying at least two of the three products $-32m$, $80n$, r that give x^{10} terms
	Coeff. of $x^{10} = (-32)(84)+(80)(14)+r$ $= -2688+1120+r$ $= -1568 + r$	**A1F**		**Only** ft c's value of r in **(a)**. If not shown in any of these forms, can be implied by final answer which matches correct evaluation of $(-1568+$c's $r)$
	Coeff. of $x^{10} = -1648$	**A1**	**5**	-1648 or left as '$-1648x^{10}$'. Ignore other powers of x terms
	Total		**8**	

Q8	Solution	Mark	Total	Comment
(a) (i)	$\frac{4\sin x}{\cos x}+\frac{5\cos x}{\cos x}=0$; $4\tan x+5=0$	**M1**		$\frac{\sin x}{\cos x}=\tan x$ clearly used to obtain a linear equation in $\tan x$.
	$\tan x=-\frac{5}{4}$ $(=-1.25)$	**A1**	**2**	-1.25 OE **NMS** mark as **B2** or **B0**
(a)(ii)	$\tan x=1$, $\tan x=-1.25$	**B1F**		1 and c's answer to **(a)(i)** vals for tan (x) PI by a correct angle for both tan values
	$(x=)$ 45°, 225°, 129°, 309°	**B2, 1**	**3**	**B2** 45, 225, AWRT 129, AWRT 309 If not **B2** award **B1** for at least two correct. If more than four values in given interval, deduct 1 mark for each extra to min of B0. Ignore values outside $0^\circ \le x \le 360^\circ$
(b)	$\frac{16+9\sin^2\theta}{5-3\cos\theta}=\frac{16+9(1-\cos^2\theta)}{5-3\cos\theta}$	**M1**		Replacing $\sin^2\theta$ by $1-\cos^2\theta$ in given expression or replacing $\cos^2\theta$ by $1-\sin^2\theta$ in term $\pm 3q\cos^2\theta$.
	$=\frac{(5-3\cos\theta)(5+3\cos\theta)}{5-3\cos\theta}$	**A1**		Or any two of $5p-3q=16$, $5q-3p=0$, $3q=9$
	$=5+3\cos\theta$	**A1**		CSO. Or q=3, p=5 **and** checking remaining eqn is satisfied.
	Least possible value is 2 and occurs at $\theta=\pi$	**B1F**	**4**	Ft on c's p and q non zero values. {If q>0, least val=$p-q$ $\theta=\pi$ } {If q<0, least val=$p+q$ at $\theta=0$ } Ignore values of θ outside given interval
	Total		**9**	

(a)(i)	$\tan x=-1.25$ OE with no errors seen scores 2 marks. Methods involving squaring and $\tan x\ne -1.25$ OE, must give reasons for discounting certain solns before M1A0 scored
(b)	Multiplying the numerator and denominator of the given expression by $5+3\cos\theta$ does not score until the correct relevant identity has been used (M1); the 1st A1 will then be awarded when the rational expression has been written in a correct form with terms which can be cancelled legitimately e.g. $\frac{(16+9\sin^2\theta)(5+3\cos\theta)}{(16+9\sin^2\theta)}$

Q9	Solution	Mark	Total	Comment
(a)	$c = 3^m$, $d = 27^n$	M1		Either $c = 3^m$ or $d = 27^n$ seen or used
	$d = 3^{3n}$, $d^2 = 3^{6n}$	A1		Either $d = 3^{3n}$ or $d^2 = 3^{6n}$ seen or used
	$\sqrt{c} = 3^{0.5m}$	A1		$\sqrt{c} = 3^{0.5m}$ seen or used
	$\frac{\sqrt{c}}{d^2} = 3^{\frac{m}{2}-6n}$	A1	4	$\frac{\sqrt{c}}{d^2} = 3^{\frac{m}{2}-6n}$ OE expression for *y* in terms of *m* and *n*.
Altn	$\frac{1}{2}\log_3 c - 2\log_3 d = y\log_3 3$	(M1)		A correct expression in *y* in terms of logs to base 3 or base 27 where no further log laws are required
	$\log_3 d = \frac{\log_{27} d}{\log_{27} 3}$	(A1)		$\log_3 d = \frac{\log_{27} d}{\log_{27} 3}$ or $\log_{27} c = \frac{\log_3 c}{\log_3 27}$ seen or used
	$\log_{27} 3 = \frac{1}{3}$	(A1)		$\log_{27} 3 = \frac{1}{3}$ or $\log_3 27 = 3$ seen or used
	$y = \frac{1}{2}m - 6n$	(A1)	(4)	Correct expression for *y* in terms of *m* and *n* OE eg $\frac{\sqrt{c}}{d^2} = 3^{\frac{m}{2}-6n}$
(b)	$1 = \log_4 4$	B1		$1 = \log_4 4$ seen or used at any stage.
	$\log_4(2x+3)(2x+15) = 1 + \log_4(14x+5)$ $\log_4(2x+3)(2x+15) = \log_4 4(14x+5)$	M1		Applying a log law correctly to two correct log terms. [Condone missing base] NB: Lots of other possibilities after correct rearrangements! PI by $(2x+3)(2x+15) = 4(14x+5)$ OE with no errors seen
	$(2x+3)(2x+15) = 4(14x+5)$	A1		OE eqn with logs eliminated in a correct manner
	$4x^2 + 36x + 45 = 56x + 20$ $4x^2 - 20x + 25 = 0$; $(2x-5)^2 = 0$ Only one solution 2.5	A1	4	Must include statement and correct value
	Total		8	
(b)	……… $4x^2 - 20x + 25 = 0$, $b^2 - 4ac = 400 - 400 = 0$, only one soln (which is $-\frac{b}{2a}$) 2.5			